

Math 241 Linear Algebra

The Magic Carpet Ride

Hex is an 18 year old hexagon who lives in the vast plane of Flatland. As she is getting ready to go to college to study Geometry with Professor Gauss, her first task is to find Gauss' office at the University of Flatland, which is located 107 miles East and 64 miles North of her parents' house. Before her parents send her off to college, they give her a magic carpet and a hover board as two means of transportation. Her father, a pentagon, explains: "Here's how it works: every time you use the magic carpet for one minute, it takes you in a straight line to a point 1 mile East and 2 miles North of where you started. Every time you get on the hover board for one minute, it takes you in a straight line to a point 3 miles East and 1 mile North of where you started". By this he means that if the magic carpet traveled forward for one minute, it would move along a diagonal path that would result in a displacement of 1 mile East and 2 miles North of its starting location. You may denote this restriction of the magic carpet's motion by writing $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Similarly, denote the hover board's motion by $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

1. Help Hex find Professor Gauss' office! Describe in detail how she may use the magic carpet and the hover board to get to his office.
2. Hex arrives at Gauss' office, but he is not there. As she picks up the magic carpet and the hover board, she discovers a hidden lever and a button on each of them with a tiny sign next to them that reads: "Pull lever to come to a full stop immediately (hang on tight!). Push button for reverse." Is there anywhere in Flatland Gauss can hide so Hex cannot get to him? Explain why or why not.
3. After a few minutes of playing with the new functions of the magic carpet and the hover board, Gauss' office neighbor, Professor Legendre, appears and gives Hex a hint: "If you're looking for grumpy Gauss, I think he's hiding from his students at his cabin these days. It's located 14.5 miles West and 7.2 miles South of here." Help Hex find Gauss' cabin! Give her detailed directions to the cabin.
4. Finally, Hex arrives at Gauss' cabin. He reveals to her that Flatland is really just a county of Sphereland and says: "See for yourself. Get into the Eloquent Elevator, and it will take you in a straight line to a point 2 miles East, 1 mile North, and 1 mile Upward." Again, by this he means that if the Eloquent Elevator traveled forward for one minute, it would move along a diagonal path that would result in a displacement of 2 miles East, 1 mile North and 1 mile Upward of its starting location. Hex gets into the Eloquent Elevator, and a pleasant voice says "your coordinates have been reset to zero, zero, zero. Brake and reverse are available for use at any time." Before she takes off, Gauss says "take a look around, and use the magic carpet, the hover board, *and* the eloquent elevator to return. Good luck!" Can you make a journey that begins and ends at Gauss' cabin using all three modes of transportation?
5. After Hex returns, there is a malfunction in the hover board that makes it move in a straight line to a point 2 miles East and 4 miles North of where she started. Is there anywhere in Flatland Gauss can hide from Hex now? Explain why or why not.

Instructions:

- 1) Write up your solutions to the 5 questions by Monday September 3. Please include detailed explanations of your answers to receive full credit!

- 2) Once we have introduced the mathematical notions of *linear combination*, *linear independence* and *span*, return to the 5 questions and explain your previous answers in terms of these new concepts. Please include your complete reasoning to receive full credit! I will officially make this an assignment when we cover the appropriate information.