Lab 1: Introduction to `Mathematica`

Unbreakable rules of `Mathematica`

- Save early and save often.

- `Mathematica` will only evaluate an input if you press `shift-enter`

- All built-in constants and functions are capitalized - some examples `Sin,Cos,Tan,E,Pi,Integrate`. If a command is a sort of compound word both parts are capitalized - for example `ArcTan`.

- (), [], and {} are all **different!!**

    - () are used **only** for grouping.
    - [] are used **only** to enter things into function (either native or user defined).
    - {} are used **only** for lists – this includes building vectors and matrices.

- Don't change outputs - don't click on an output and type things. This is very confusing to `Mathematica` and will hurt its feelings.

Arithmetic

We can use `Mathematica` to do simple arithmetic. For example if we want to find $\cos \frac{\pi}{4} + \sqrt{31}$ we would use the following

$$\text{In[198]:= } \textbf{Cos[Pi / 4] + Sqrt[31]}$$

$$\text{Out[198]= } \frac{1}{\sqrt{2}} + \sqrt{31}$$

If we wanted to force a decimal expansion of this we could use the `N` command:

$$\text{In[199]:= } \textbf{N[Cos[Pi / 4] + Sqrt[31]]}$$

$$\text{Out[199]= } 6.27487$$

Defining constants

There are three slightly different methods for assigning values to constants in `Mathematica`

- Simple assignment: The following assigns the value of 3.24 to `a` and prints this assignment.

$$\text{In[201]:= } \textbf{a = 3.24}$$

$$\text{Out[201]= } 3.24$$

- Simple assignment without output. Sometimes we don't need to print the assignment we just made:

$$\text{In[202]:= } \textbf{a = 3.24;}$$

- Delayed assignment. This type of assignment delays any calculation until `Mathematica` needs this object.

$$\text{In[203]:= } \textbf{a := 3.24}$$

The following code defines values for a, b, and c and performs the calculation $ab + a^2 - \frac{1}{c} + e^b$. We can push `enter` between each line and then evaluate everything at the end by using `shift-enter` this keeps things a little tidier.

```
In[208]:= a = N[Sin[1]];
          b = 2.5;
          c = 42;
          a * b + a ^ 2 - (1 / c) + E ^ b
```

Out[211]= 14.9704

Notice that we carefully used semi-colons to suppress the output when it was not needed.

Working with functions

Defining functions

We have the same three methods for defining functions with the important difference that we must tell `Mathematica` what the variables are.

- Simple assignment: The following defines the function $f(x) = x^2 \ln x$ and prints this assignment.

```
In[212]:= f[x_] = x ^ 2 * Log[x]
```

Out[212]= x^2 Log[x]

- Simple assignment without output. Sometimes we don't need to print the assignment we just made:

```
In[213]:= f[x_] = x ^ 2 * Log[x];
```

- Delayed assignment: There are times when `Mathematica` will have errors unless you define your functions this way:

```
In[259]:= f[x_] := x ^ 2 * Log[x]
```

- Functions with more than one variable.The following function takes two numbers and finds their sum.

```
In[23]:= P[m_, n_] := m + n
         P[2, 3]
```

Out[24]= 5

Remark.

- We only use the notation `x_` when *defining* a function. Never when using the function.

- I will often give you examples of functions that you have to slightly change. It is highly suggested that you follow my method of definition carefully, including the choice of simple assignment or delayed assignment.

Using Functions:

Now we can use this function to do a number of things.

- Lets evaluate it at a point - find $f(4.2)$

```
In[261]:= f[4.2]
```

Out[261]= 25.3149

- We can also do a simple composition like $f(x^2 + y^3)$:

```
In[262]:= f[x ^ 2 + y ^ 3]
```

Out[262]= $\left(x^2 + y^3\right)^2$ Log$\left[x^2 + y^3\right]$

- Lets take a derivative

- and a second derivative

Some Linear Algebra

Solving Systems of Equations

There are a few ways we can use `Mathematica` to solve equations, but for our uses we only need to worry about one. Before we look at these methods lets notice that `Mathematica` uses `=` for assignment and `==` to set up and equation.

- A consistent solution with one solution. Consider the system of equations given by

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 - 3x_2 + 5x_3 = 11$$
$$x_1 - x_2 + x_3 = 2.$$

We can solve this using `Mathematica` as follows:

In[10]:= **Solve[{x1 + x2 + x3 == 6, 2 * x1 - 3 * x2 + 5 * x3 == 11, x1 - x2 + x3 == 2}, {x1, x2, x3}]**

Out[10]= {{x1 → 1, x2 → 2, x3 → 3}}

- A consistent solution with infinitely many solutions. Consider the system of equations given by

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 - 3x_2 + 5x_3 = 11$$

We can solve this using `Mathematica` as follows:

In[11]:= **Solve[{x1 + x2 + x3 == 6, 2 * x1 - 3 * x2 + 5 * x3 == 11}, {x1, x2, x3}]**

⋯ Solve: Equations may not give solutions for all "solve" variables.

Out[11]= $\left\{\left\{x2 \to \dfrac{19}{8} - \dfrac{3\,x1}{8}, x3 \to \dfrac{29}{8} - \dfrac{5\,x1}{8}\right\}\right\}$

Observe that `Mathematica` takes x_1 as the free variable. This is different than, but equivalent to, the standard we have been using in class.

- An inconsistent solution. Consider the system of equations given by

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 - 3x_2 + 5x_3 = 11$$
$$x_1 - x_2 + x_3 = 2$$
$$x_1 + x_2 = 4.$$

In[13]:= **Solve[{x1 + x2 + x3 == 6, 2 * x1 - 3 * x2 + 5 * x3 == 11, x1 - x2 + x3 == 2, x1 + x2 == 4},**
{x1, x2, x3}]

Out[13]= {}

Solving Matrix Equations

Recall from class that a system of equations is equivalent to a matrix equation. For example the system of equations

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 - 3x_2 + 5x_3 = 11$$
$$x_1 - x_2 + x_3 = 2.$$

is equivalent to the matrix equation

$$A\mathbf{x} = \mathbf{b}$$

where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 5 \\ 1 & -1 & 3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 6 \\ 11 \\ 2 \end{pmatrix}.$$

In order to solve this system using `Mathematica` we will first define the matrix A and vector $\mathbf{b}$ as follows.

In[7]:= **A = {{1, 1, 1}, {2, -3, 5}, {1, -1, 1}}**

Out[7]= {{1, 1, 1}, {2, -3, 5}, {1, -1, 1}}

The output form doesn't look like the way we have been writing matrices. In order to force `Mathematica` to make a more familiar output we can do the following:

In[8]:= **MatrixForm[A]**

Out[8]//MatrixForm=
$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 5 \\ 1 & -1 & 1 \end{pmatrix}$$

It is always good to use this command in order to check that you entered the matrix correctly. Now we define the vector $\mathbf{b}$.

In[3]:= **b = {6, 11, 2}; MatrixForm[b]**

Out[3]//MatrixForm=
$$\begin{pmatrix} 6 \\ 11 \\ 2 \end{pmatrix}$$

Now we can solve this matrix equation using the `LinearSolve` command and simultaneously set the solution equal to the vector $\mathbf{x}$

In[9]:= **x = LinearSolve[A, b]; MatrixForm[x]**

Out[9]//MatrixForm=
$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Finally we can check this solution by performing matrix-vector multiplication using `Mathematica`. Notice that, here we use a period "." as the multiplication operator.

In[11]:= **MatrixForm[A.x]**

Out[11]//MatrixForm=
$$\begin{pmatrix} 6 \\ 11 \\ 2 \end{pmatrix}$$

Other Matrix Operations

We will now explore some other matrix operations in `Mathematica`. We will first define two matrices to work with in this section.

$$A = \begin{pmatrix} 1 & 8 & -1 \\ 4 & 5 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 7 & -10 \\ 2 & 1 \\ -3 & 6 \end{pmatrix}.$$

In[22]:= **A = {{1, 8, -1}, {4, 5, 3}};**
MatrixForm[A]

Out[23]//MatrixForm=
$$\begin{pmatrix} 1 & 8 & -1 \\ 4 & 5 & 3 \end{pmatrix}$$

In[24]:= **B = {{7, -10}, {2, 1}, {-3, 6}};**
MatrixForm[B]

Out[25]//MatrixForm=
$$\begin{pmatrix} 7 & -10 \\ 2 & 1 \\ -3 & 6 \end{pmatrix}$$

Matrix Addition and scalar multiplication

There are no tricks to matrix addition and scalar multiplication.

In[29]:= **MatrixForm[2 * A + A]**

Out[29]//MatrixForm=
$$\begin{pmatrix} 3 & 24 & -3 \\ 12 & 15 & 9 \end{pmatrix}$$

What is interesting is that if you attempt to add matrices of different sizes `Mathematica` will not cooperate:

In[28]:= **MatrixForm[A + B]**

⋯ Thread: Objects of unequal length in {{1, 8, −1}, {4, 5, 3}} + {{7, −10}, {2, 1}, {−3, 6}} cannot be combined.

Out[28]//MatrixForm=
{{1, 8, -1}, {4, 5, 3}} + {{7, -10}, {2, 1}, {-3, 6}}

Matrix Multiplication

As described above, `Mathematica` used a period "." to perform matrix multiplication. Here are a few examples:

In[26]:= **MatrixForm[A.B]**

Out[26]//MatrixForm=
$$\begin{pmatrix} 26 & -8 \\ 29 & -17 \end{pmatrix}$$

In[27]:= **MatrixForm[B.A]**

Out[27]//MatrixForm=
$$\begin{pmatrix} -33 & 6 & -37 \\ 6 & 21 & 1 \\ 21 & 6 & 21 \end{pmatrix}$$

Finding Help

There is a wealth of information online that you can use to learn `Mathematica`. I would suggest by starting with this video (`http://www.youtube.com/watch?v=ugRuESa_cu0`) and exploring others as needed. I would suggest watching the videos "What's new in Mathematica 9" as well as "Mathematica 8: Using the Free-form input and WolframAlpha data". In addition the `Mathematica` help is very useful. If you know the name of the command you can simple type it into `Mathematica` preceded by a `?` to learn all you need to know.

In[4]:= `? Simplify`

> Simplify[expr] performs a sequence of algebraic
>   and other transformations on expr, and returns the simplest form it finds.
> Simplify[expr, assum] does simplification using assumptions. ≫

If you then want to learn more about this command you can click on the `>>` on the bottom left of the help box. This will open up the full `Mathematica` help document for this command which contains examples, applications, and more. If you are not sure of the name of the command needed open the *Documentation Center*, found in the help menu. Here you can easily explore all `Mathematica` has to offer.

Writing up Labs

You will be turning in your solutions to these labs in your groups. Here are a few expectations to keep in mind when preparing your lab write up.

- Between `Mathematica` commands you should document what you are doing in complete sentences. If you hover the mouse between commands the cursor will be horizontal. At this point you can click and then choose `Format`→`Style`→`Text` in the top menu. In this mode `Mathematica` behaves as a word processor. Feel free to experiment with other types of formatting styles as you see fit. It may be especially nice to use inline equations - more typesetting tools may found in the *Classroom Assistant* pallet.

- You should write your solutions so that a peer (another Linear Algebra student) is able to follow your process.

  Here is an example of what I am looking for:

  - **Question 1:**

    Find the equation of the tangent line of f(x)=arctan$\left(\frac{x^2-1}{x^2+1}\right)$ at x=2. Plot the curve and it tangent line on the same set of axes.

∎ **Solution:**

We will begin by defining the function f(x) and assigning the value of a=2 (this is optional).

```
In[10]:= f[x_] := ArcTan[(x^2 - 1) / (x^2 + 1)]
         a = 2;
```

Now we will use the derivative of f(x) at x=2 to find the slope of the tangent line - we assign this value to m.

```
In[12]:= m = D[f[x], x] /. {x → a}
```

$$Out[12]= \frac{4}{17}$$

In order to find the equation of the tangent line we also need the y-coordinate of the point. We will assign this value to the letter b.

```
In[14]:= b = f[a]
```

$$Out[14]= ArcTan\left[\frac{3}{5}\right]$$

If we want to express the point (a,b) and slope m as a decimal we can use the N- command. Recall that *Mathematica* uses {} for lists and considers a coordinate to be a list.

```
In[16]:= {N[a], N[b]}
         N[m]
```

```
Out[16]= {2., 0.54042}
```

```
Out[17]= 0.235294
```

Now we can use the point-slope form of a line to write the equaiton of the tangent - here we use the DisplayFormula Style. I have changed the size, text color, and background color so that the solution stands out.

$$y - 0.54042 = 0.235294 \ (x - 2)$$

We can perform some aritmatic to put this equation in a more familar form:

$$y = 0.235294 \ x - 0.069832$$

Now we will define a function T in order to plot this line - notice that here we make liberal use of our predefied constants a, b, and m.

```
In[18]:= T[x_] := m (x - a) + b
```

All that is left is to plot the curve and its tangent line on the same graph, which we do below. (Make sure and play around with the maximum and minimum x-values until you have a nice piture.

```
In[22]:= Plot[{f[x], T[x]}, {x, -3, 6}, PlotStyle → {Thick, Thick}]
```

Out[22]=
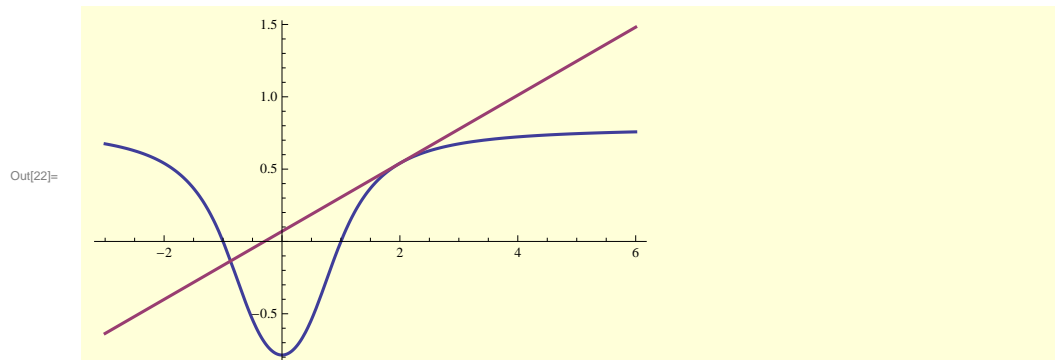
- You can turn in labs by first saving the file as a pdf – choose `File→SaveAs` and you will be given that choice. Then you can email me the file. Make sure and give both the file and the subject line of the email a distinguishing title so that I can distinguish it from everyone else's lab.

Questions

This lab is due at the start of class on Wednesday, December 5. Please print the notebook and hand it in. It will help visually helpful to nest many of your commands in the `Mathematica` function `MatrixForm[...]` (like I did in the above examples) so that everything is output looking like a matrix and is more familiar.

1. Consider the system of equations

$$3x_1 - 6x_2 - x_3 + x_4 = 6$$
$$-x_1 + 2x_2 + 2x_3 + 3x_4 = 3$$
$$4x_1 - 8x_2 - 3x_3 - 2x_4 = 3.$$

   (a) Use the `Solve` command to solve this system of equations.

   (b) Use the `LinearSolve` command to solve the matrix equation associated to this system of equations.

   (c) Check your result from part (b) using matrix multiplication in `Mathematica`.

2. Consider the system of equations

$$x_1 + x_2 + x_3 = 1$$
$$2x_1 - x_2 + 5x_3 = 5$$
$$-4x_2 - x_3 = 4$$

   (a) Use the `Solve` command to solve this system of equations.

   (b) Use the `LinearSolve` command to solve the matrix equation associated to this system of equations.

   (c) Do a search (google or `Mathematica` help) to discover how to find the inverse of a matrix using `Mathematica`. Solve this system of equations using matrix multiplication in `Mathematica` with the inverse of the appropriate matrix.

3. Consider the matrices

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & 7 \\ 1 & 2 & 4 \end{pmatrix}, B = \begin{pmatrix} 7 & 3 & -6 \\ 10 & 13 & 9 \\ 4 & 2 & 1 \end{pmatrix}, X = \begin{pmatrix} 1 & 9 & -5 & 0 \\ 2 & 7 & 11 & -1 \\ 3 & 10 & 0 & -3 \end{pmatrix}, \text{ and } Y = \begin{pmatrix} 5 & 1 & -4 \\ -13 & -1 & 5 \\ 9 & 2 & 8 \\ 6 & -3 & 1 \end{pmatrix}.$$

   (a) Do a search to discover how to find the transpose of a matrix using `Mathematica`. Find the transpose of the matrices $A, B, X,$ and Y.

   (b) Do a search to discover how to find the row reduced echelon form of a matrix using `Mathematica`. Find the row reduced echelon form of the matrices $A, B, X,$ and Y.

   (c) Calculate
$$AB^{-1} + 2BA - 3(XY)^{-1}.$$

   (d) The commutator of two matrices, C and D, is defined by

$$[C, D] = CD - DC.$$

   i. Write a `Mathematica` function that will calculate the commutator of two matrices – call it `MatrixCommutator`. Hint: You may want to use a delayed function definition as describe above.

   ii. Using the matrices defined above, use the function you defined in part a to find:

$$[A, B] \text{ and } [AB, XY].$$