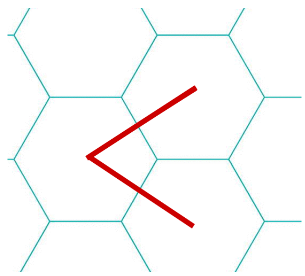
Modeling a Zombie Outbreak

Creating a model

Humans and zombies live on a hexagonal grid and move around randomly with the exception that no two humans or zombies may occupy the same cell at any given time. A human may occupy the same cell as a zombie, however. Each day at midnight every zombie on the grid will attack any human that happens to be in its own cell as well as the two cells that it can reach with its arms as described in the below image.



Question 1. Suppose there is one zombie and one human on a grid with 20 hexagons. What is the probability that the human will become infected after one day?

Question 2. Suppose there is one zombie and ten humans on a grid with 20 hexagons. What is the expected number of humans that will be newly infected at the end of the first day?

Question 3. Suppose there are five zombies and one human on a grid with 20 hexagons. Assuming that there is no overlap of the cells that each zombie will attack, what is the probability that the human will be infected after one day?

Question 4. Suppose there are five zombies and ten humans on a grid with 20 hexagons. Assuming that there is no overlap of the cells that each zombie will attack, what is the expected number of humans that will be newly infected at the end of the first day?

Now we can put together what we have explored in the previous four questions into a word equation that describes the rate at which humans are infected.

$$\begin{pmatrix}\text{change in}\\ \text{number of zombies}\end{pmatrix} = \frac{???}{\text{total number of cells}}\left(\text{number of zombies}\right)\left(???\right) \qquad (1)$$

If we set

$$Z = \left(\text{number of zombies}\right) \text{ and } H = \left(\text{number of humans}\right),$$

we can rewrite (1) as a differential equation

$$Z' = ???$$

Now we want to apply this to build a model for a situation where 49 humans and 1 zombie are placed onto the grid you have been given.

Exercise 1. Write a differential equation that describes the expected change in number of zombies in this situation.

Exercise 2. Notice that the total population of zombies and humans will remain constant. Use this to rewrite your differential equation so that it only contains the variable $Z =$ the population of zombies. Make sure to include an initial condition.

Now we have built a model for our zombie outbreak. Before we solve this initial value problem we will simulate the zombie outbreak and collect data.

Data Collection

1. Begin with a population of 49 humans and 1 zombie and place them randomly on separate grids.
2. Overlap the grids and record the new number of zombies and humans according to the infection rule.
3. Erase both grids.
4. Repeat steps 3-5 with the new population sizes until all humans have been infected.
5. Enter your data in the shared spreadsheet for the class:

   `goo.gl/amXGpp`

6. Define a list in `Mathematica` for your groups data as well as the average data from the class. The list should look something like:

   ```
   ZombiePopulation={{0,1},{1,4},{2,8},...{10,50}};
   ```

7. Define plots for each of these data sets. This should look something like:

   ```
   Pzombie=ListPlot[ZombiePopulation]
   ```

8. Plot your groups data against the class average. What do you notice? Was your outbreak quicker or slower?

Comparing the model with the data

Exercise 3. Recall the initial value problem that you constructed to model this zombie outbreak. Solve this differential equation three ways:

1. By hand, using separation of variables and integration via partial fraction decomposition.
2. Using the `DSolve` command in `Mathematica`.
3. Numerically approximating a solution with Euler's method using a loop in `Mathematica` with 1000 steps.

Exercise 4. Check that your exact solution that you calculated by hand coincides with the solution you found in `Mathematica`.

Exercise 5. The exact solution and the numerical approximation via Euler's method on the same axis.

Exercise 6. Plot the exact solution to the modeling differential equation alongside the data your group generated. How well does the model fit the data?

Exercise 7. Plot the exact solution to the modeling differential equation alongside the class average data. How well does the model fit the data?

Exercise 8. Can you think of anything that may reduce the preciseness of this model?

The written report

Each group should turn in one report. This report should address the above exercises in the text without explicitly stating them. Your report should have the following sections.

1. Abstract
2. Background on the logistic model – this is the differential equation that you built. You can find background on this model in general in the text.
3. The set-up of the experiment and initial data presentation.
4. Constructing a differential equations model of this experiment.
5. Discussion: Strengths and weaknesses of the model, possible improvements.