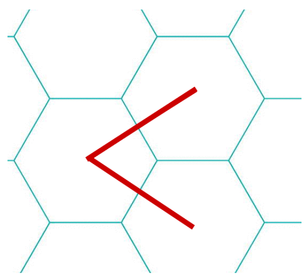
Modeling a Zombie Outbreak

Creating a model

Humans and zombies live on a hexagonal grid and move around randomly with the exception that no two humans or zombies may occupy the same cell at any given time. A human may occupy the same cell as a zombie, however. Each day at midnight every zombie on the grid will attack any human that happens to be in its own cell as well as the two cells that it can reach with its arms as described in the below image.



Assume an initial zombie population of 1 and human population of 49, the last time we performed this experiment we built the intuitive equation

$$\begin{pmatrix}\text{change in}\\ \text{number of zombies}\end{pmatrix} = \frac{\text{number of cells each zombie infects}}{\text{total number of cells}}(\text{number of zombies})(\text{number of humans}),$$

which became the differential equation

$$Z' = kZH,$$

where $H = 50 - Z$. This is equivalently a differential equation for the change in the human population

$$H' = -kZH. \tag{1}$$

This gave us the initial value problem

$$Z' = kZ(50 - Z) \text{ with } Z(0) = 1,$$

which we were able to solve exactly.

This time we will make a different assumption – that after three days of being infected each zombie recovers. In this state, the individual cannot become infected or transmit the illness. We may break the population into the following groups

$$\begin{aligned} S =&\ \text{susceptible individuals,}\\ I =&\ \text{infected individuals (aka Zombies!!)}\\ R =&\ \text{recovered individuals} \end{aligned}$$

Exercise 1. We will again assume the total population is 50. Use this to write a very simple equation (not a differential equation) that involves S, I, and R.

Exercise 2. Our next goal is to build a model for this outbreak. Use the following guidelines to write down a system of differential equations.

1. Take inspiration from (1) to write down a differential equation for the change in the susceptible population
$$S' = \underline{\qquad\qquad}.$$
2. Use the fact that the infected population recovers in three days to write down a differential equation for the change in the recovered population
$$R' = \underline{\qquad\qquad}.$$
3. Use the fact (from Exercise 1) that $S' + I' + R' = \underline{\qquad}$ as well as the previous two exercise to write down a differential equation for the infected population
$$I' = \underline{\qquad\qquad}.$$

Now we have built a model for our zombie outbreak. Before we consider the solution of this system of differential equations, we will simulate the zombie outbreak and collect data.

Data Collection

1. Begin with a population of 49 humans and 1 zombie and place them randomly on separate grids.
2. Overlap the grids and record the new number of zombies and humans according to the infection rule.
3. Erase both grids.
4. Repeat steps 3-5 with the new population sizes until all zombies have recovered.
5. Enter your data in the shared spreadsheet for the class (in Sheet 2):

   `goo.gl/amXGpp`

6. Define a list in `Mathematica` for your groups data as well as the average data from the class. The list should look something like:

```
SusceptiblePopulation={{0,1},{1,4},{2,8},...{10,50}};

InfectedPopulation={{0,49},{1,46},{2,42},...{10,0}};

RecoveredPopulation={{0,0},{1,0},{2,0},...{10,50}};
```

7. Define plots for each of these data sets. This should look something like:

```
PInfected=ListPlot[InfectedPopulation]
```

8. Plot your groups data against the class average. What do you notice? Was your outbreak quicker or slower?

Comparing the model with the data

Exercise 3. Recall the system of differential equations that you constructed to model this outbreak. This system is not exactly solvable so we need to use the `NDSolve` command in Mathematica.

1. Enter `?NDSolve` to read about the syntax for solving a system of differential equations using this command.

2. Using the `NDSolve` command in `Mathematica` produce a "solution" to this differential equation.

Exercise 4. Plot the solution to the modeling differential equation alongside the data your group generated. How well does the model fit the data?

Exercise 5. Plot the exact solution to the modeling differential equation alongside the class average data. How well does the model fit the data?

Exercise 6. Can you think of anything that may reduce the preciseness of this model?

The written report

Each group should turn in one report. This report should address the above exercises in the text without explicitly stating them. Your report should have the following sections.

1. Abstract

2. Background on the SIR model – this is the differential equation that you built. There is a bit of information in the text, but you should also have at least one outside source.

3. The set-up of the experiment and initial data presentation.

4. Constructing a differential equations model of this experiment.

5. Discussion: Strengths and weaknesses of the model, possible improvements.