Modeling Death and Migration

Adapted from:
Brian Winkel (2015), "1-001-S-MandMDeathAndImmigration," https://www.simiode.org/resources/132.

Set-up

For this scenario you are supplied with approximately 50 pennies in a small container, each of which represent a member of a certain population.

The experiment

Shake the pennies out onto your table and separate them into those that are "heads" and those that are "tails". If a penny is "heads" it dies and is removed from the population. Count the number of individuals that survive and put them back into the container. Put those that die aside. Repeat this until no-one survives.

1. Can you make a prediction about what should happen? Guess the population after each step.
2. In the following table write your guess for the population at each step as well as the actual data from completing the experiment.

Modeling Death

Step	Guess Population	Actual Population
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

3. Compare your guess with the data. Why was your guess good? Could it have been improved?

Our first mathematical model

4. Set $P(t) =$ the population at time t. So $P(0) =$??. After one step we expect the population to be approximately what proportion, k, of the population at the previous step? As an equation this is
$$P(t+1) = kP(t).$$

5. Since we want to find a continuous model we need to find how the population has changed after h-units of time. Explain why
$$P(t+h) = k^h P(t), \tag{1}$$
for the same value of k that you decided on above.

6. Recall that the derivative is defined at
$$\frac{dP}{dt} = \lim_{h\to 0} \frac{P(t+h) - P(t)}{h}.$$
Use (1) to find an expression
$$\frac{dP}{dt} = \_\_\_\_\_.$$
Hint: You may want to use L'Hospital's rule for the limit, where the identity $\frac{d}{dx}a^x = (\ln a)a^x$ may be helpful.

7. Solve the differential equation using the initial condition. How does your model match your data?

Death and immigration model

Repeat the above experiment except in addition to a portion of the population dying add 10 new individuals at each step. Guess the outcome of this experiment.

1. Record your data in the following table:

Modeling Death and Immigration

Step	Guess Population	Actual Population
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

2. Repeat the steps above to form a mathematical model of this scenario and compare your model with the data.